Solving Optimal Groundwater Problems with Excel

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Abstract

Welfare maximizing management of coastal groundwater requires a sequence of pumping targets, typically terminating with a constant withdrawal rate in the long run. In contrast, managing according to sustainable yield at best identifies the constant rate of pumping in the long run. We illustrate an accessible solution method, using Excel Solver to find the optimal transition paths of groundwater pumping, price, and head level and the corresponding solutions in the long run. The developed framework is applied to the Pearl Harbor Aquifer (PHA) in Hawaii using data from previous studies. Each step of the solution process is described, including setting parameter values and defining objective, variable, and constraint cells in Excel to facilitate successful replication of the results. Possible extensions are also discussed such as watershed conservation, protection of groundwater dependent ecosystems, and management of multiple aquifers.

1 Introduction

Groundwater depletion and degradation, exacerbated by climate change, are increasingly recognized as critical issues in many groundwater-dependent parts of the world. Over time a typical aquifer is recharged naturally from precipitation that infiltrates below ground and can also be recharged via irrigation return flow, due either to canal leakage or excess applied water not consumed by crops. The cost of withdrawing groundwater is a function of lift, or the distance between the water table and the surface. The relevant management problem is to determine how much groundwater to withdraw over time, taking into account whenever possible the interlinkages between the aquifer, the watershed, and groundwater-dependent ecosystems.

Groundwater resources account for 99 percent of Earth's freshwater,[1] yet only a fraction of this is accessible without exceedingly large pumping costs and without risking salinization or over-depletion. Groundwater provides for all the daily water needs for one third of the world's population and is the only source of freshwater for all human needs in many parts of the world, particularly in remote and dry areas (World Bank 2016). Groundwater provides a buffer to climatic variability, acts as storage during droughts, contributes to river flow, and supports near shore marine ecosystems. Global groundwater withdrawals have more than quadrupled in the last 50 years. This extraction uses significant amounts of energy, although because energy is often heavily subsidized, the true costs of groundwater extraction are misunderstood, unknown, or ignored completely. While resource economics provides the tools to make the full value of groundwater transparent, so far there has not been adequate coverage of this topic in textbooks and many classrooms, especially as compared to nonrenewable resources such as oil and coal, or renewable resources that traditionally command a tradeable price in the market such as fish and forest products. Without a clear understanding of the efficient extraction and pricing of

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[1] For a more detailed accounting of the Earth's water resources, including ice and permafrost, see Water Science School (2018).

groundwater, students (and practitioners) are left with picking a long run target such as maximum sustainable yield (MSY) and an inevitably ad hoc and arbitrary approach path, which will further exacerbate the global water scarcity crisis.

While textbooks developed for undergraduate and graduate classes in natural resource economics provide complete coverage for exhaustible resources such as coal or oil, as well as full chapters devoted to renewable resources such as fisheries and forests, the chapter on water resources is typically focused on the difference between finding efficient allocations for surface water versus groundwater, with limited details provided for optimal groundwater extraction and pricing over time (Perman, McGilvray, and Common 2003; Field 2015; Tietenberg and Lewis 2020). In this chapter we provide what is currently missing in natural resource economics textbooks to facilitate understanding of the optimality conditions for groundwater resources.

To add more complete coverage of efficient allocation and pricing for groundwater resources, this framework was used in advanced undergraduate and master's level Ecological Resource Economics classes by one of our co-authors during her sabbatical teaching at Kobe University in Japan.[2] The derivation of the Pearce equation in conjunction with the Excel example facilitated clearer understanding of optimal allocation and pricing, especially since the chapter on water in the textbook used in the course (Tietenberg and Lewis 2020) followed a detailed discussion of the optimal price equation for nonrenewable resources but provided no corresponding explanation for renewables. Seeing the marginal extraction cost (MC), marginal user cost (MUC), and marginal externality cost (MEC) components of the Pearce equation in this manuscript helped the students clearly see the connection and differences compared to the same analysis of nonrenewable resources. This is an important and missing piece to many natural resource economics textbooks currently available. Without the Pearce equation provided in this chapter, learning outcomes regarding efficient allocation and pricing of groundwater were incomplete and imbalanced compared to more in-depth coverage of nonrenewable resources or renewables such as fisheries or forests. In particular, the explanation of the MUC reflecting the intertemporal opportunity cost of groundwater is a crucial but often underappreciated component in teaching groundwater economics.

In what follows, we focus primary attention on coastal aquifers, inasmuch as inland aquifers can be analyzed as a special case. A coastal aquifer can be thought of as a freshwater lens floating on denser underlying ocean water, where the volume of groundwater in storage depends primarily on the aquifer's boundaries, porosity, and head level (Figure 1). In particular, the greater the head level, the more water is discharged into the ocean, decreasing net recharge.

In many jurisdictions, including water-challenged states such as California and Hawaii, management of groundwater relies on the principle of sustainable yield (Roumasset and Wada 2010, 2013; Elshall et al. 2020). California's Assembly Bill No. 1739, Chapter 347 defines their "sustainability goal" as "the existence and implementation of one or more groundwater sustainability plans that achieve sustainable groundwater management by identifying and causing the implementation of measures targeted to ensure that the applicable basin is operated within its sustainable yield." The bill defines "sustainable yield" as the maximum quantity of water that can be withdrawn annually from a groundwater supply without causing an undesirable result. Hawaii's groundwater regulators also base groundwater management on sustainable yield, defined as the maximum extraction rate that can be sustained without "impairing" the aquifer. The Hawaii Water Commission then exercises some discretion in determining whether a proposed increase in extraction is too close to sustainable yield.[3] A number of other states in the United States are increasingly governed by similar types of regulations,

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[2] A syllabus for the course is available in the online supplementary material.
[3] Hawaii State Water Code, Chapter 174c and L 1987, c 45, pt of §2; am L 1999, c 197 §6.
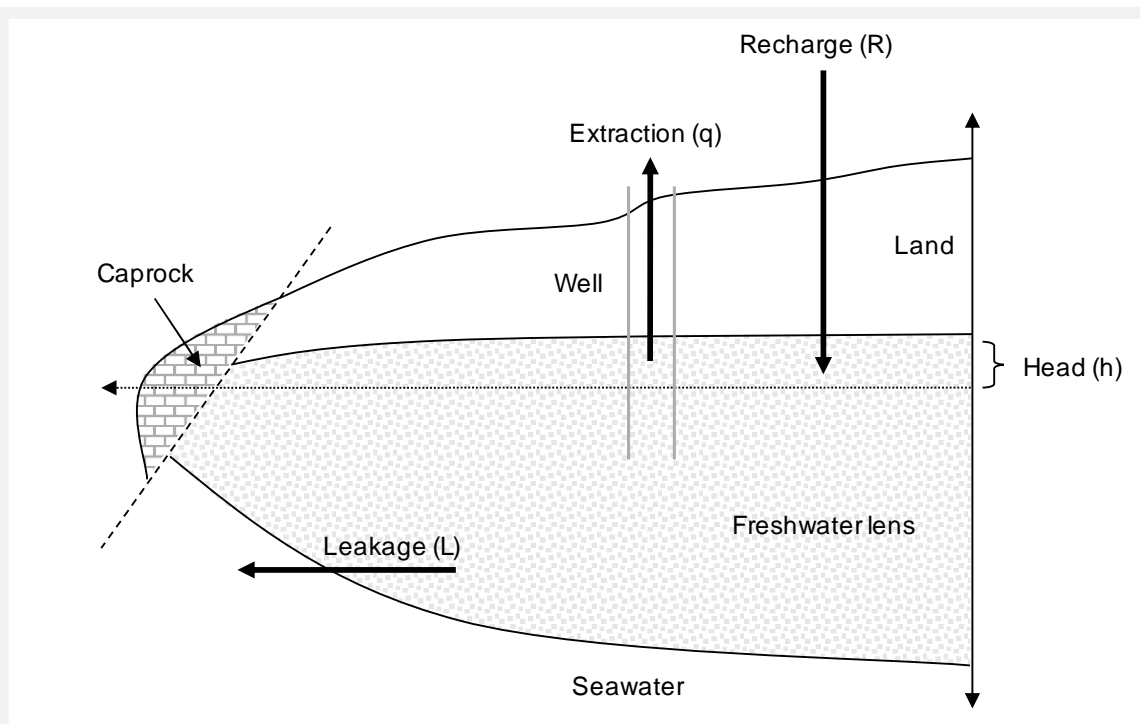
Figure 1: Cross-Section of Coastal Aquifer. Adapted from Roumasset and Wada (2012)

with Oregon defining "sustained yield" as "the amount of water that can be withdrawn from the groundwater basin annually without exceeding the long-term mean annual water supply to the reservoir"[4] and Arkansas and Louisiana both relying upon sustainable yield measures for groundwater management (Texas A&M University School of Law 2017).

The primary question for which resource economics was developed is how much to extract from a natural resource stock over time. The answer to the question is a vector. But California, Hawaii, and other jurisdictions relying on sustainable yield are implicitly searching for a scalar, as if one number could provide the answer for any number of years. Specifically, we seek the sequence of groundwater withdrawals over time that maximizes the present value (PV) of an aquifer. In some cases, the sequence of optimal withdrawals will converge to a single quantity to be extracted in the long run, called the "steady state" solution, which may or may not coincide with the stock corresponding to MSY stock. The objective of this paper is to set up a framework to solve for the optimal management of a coastal groundwater resource and illustrate how the extraction and price paths can be solved in a user-friendly and transparent Excel spreadsheet suitable for classroom use. We conclude by discussing possible extensions to the basic groundwater management problem to allow for additional challenges such as multiple water resources, consideration of linked groundwater dependent ecosystems, and improved recharge due to upstream watershed conservation.

2 Sustainable Yield Is Incomplete as a Management Strategy

Groundwater is typically viewed as a renewable resource in the sense that aquifers—subsurface layers of water-bearing permeable rock or sediment—can be replenished over time by groundwater recharge. The natural recharge rate, which is primarily determined by precipitation, is analogous to the biological

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[4] This approach embodies two fallacies. First, the existing stock, which the strategy aims to maintain, may be either too high or too low. Second, the goal is to identify a constant extraction rate when consumer welfare may require lower amounts in early years.

growth rates that characterize other renewable resources such as trees and fish. An aquifer can also be recharged by irrigation return flow due to canal leakage and/or excessive application of water to crops, as well as by inflows from adjacent and more elevated bodies of freshwater. In the case of a coastal aquifer, even in the absence of pumping for aboveground uses, groundwater naturally discharges from the aquifer at the ocean boundary. The rate of submarine groundwater discharge (SGD) along the saltwater interface is a function of the groundwater stock, inasmuch as a larger stock creates a wider surface area across which groundwater can discharge, as well as more pressure at the interface. Thus, left unharvested, the groundwater stock grows due to natural recharge, and the rate of growth depends on stock-dependent SGD. With these resource characteristics in mind, the water manager's challenge is to determine how much groundwater to pump over time.

In the past, a common management recommendation by non-economists was to limit extraction of a renewable resource to a safe yield or sustainable yield. In the context of groundwater, the former constrains pumping by the amount of recharge, while the latter takes into account discharge as well, such that the constraint is based on groundwater capture (net recharge). More recently, groundwater management in many regions has evolved toward some form of sustainable groundwater management that accounts for hydrological, environmental, and socioeconomic consequences of pumping, while incorporating stakeholder participation and adaptive governance (Elshall et al. 2020). Although much more holistic than earlier approaches to groundwater management, recent efforts still often focus on what yield can be sustained in the long run without clear recommendations on how to approach said long run yield. Only by coincidence can this approach find the extraction path that maximizes the contribution of groundwater to the general welfare.

In contrast, resource economics poses the question: "What is the approach path that maximizes the contribution of an aquifer to consumer welfare?" In the remainder of this paper, we discuss, in simple terms, the welfare maximization problem and provide an example of how to find the optimal approach path using a framework developed for Excel's Solver.

3 Optimal Extraction Is Sustainable But Not the Other Way Around

This section begins with an intuitive derivation of the governing equations for the groundwater management problem. Although not used directly in the Excel application, we provide what is currently missing in natural resource economics textbooks regarding the optimality conditions for groundwater resources, couched in relatively simple marginal benefit and marginal cost terms, for students who may be unfamiliar with dynamic optimization methods.[5] We then provide an approach to solve the standard groundwater management problem using Excel Solver. The section concludes with an application to the Pearl Harbor Aquifer (PHA) in Hawaii.

3.1 Intuitive Derivation of the Governing Equations

The standard resource economics approach to groundwater management is to maximize the PV of net benefits generated by the aquifer. The solution specifies both the optimal steady state stock level and the optimal approach path to that long run target. Although the optimal steady state level may be very close to or even coincide with the constraint specified by other groundwater management approaches, the rate of optimal pumping is typically not constant over time. Moreover, the corresponding groundwater stock level may follow an increasing, decreasing, or nonmonotonic path toward the steady state.

While there are an infinite number of candidate sequences that can potentially solve the maximization problem, we can narrow down the possibilities by imposing an *equation of motion* for the aquifer stock, which describes how the quantity of the groundwater stock (h) changes over time in response to pumping decisions in every period. The analytical solution to the water manager's problem requires a dynamic optimization approach such as optimal control, which requires proficiency in

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[5] More rigorous mathematical equations describing the optimization problem can be found in the Appendix.

differential equations. To promote accessibility, we provide an intuitive derivation of the necessary condition for maximum welfare instead. We then explain and illustrate how to use Excel Solver to obtain numerical solutions.

We start with the simplification wherein SGD is zero (e.g., an inland aquifer), and the recharge rate is very low (effectively zero over the management horizon of interest). In this case, groundwater can be treated as a nonrenewable resource and the derivation proceeds accordingly. As with many other problems, optimal extraction is governed by the *Nike rule of economics*: *Just do it until marginal benefit equals marginal cost*. In this case, it is convenient to consider the marginal benefit and cost of "waiting," that is, postponing harvest of the marginal unit by one period. The marginal benefit of waiting is $\dot{p}$, that is, the capital gain from selling at next period's price. The marginal cost of waiting is the foregone interest from harvesting today instead of one period later, that is, the real interest rate times the net price or $r(p\text{-}c)$. Dividing both sides by r and adding cost to both sides yields

$$p_t = c(h_t) + \frac{\dot{p}}{r} \tag{1}$$

Equation (1), called the "Pearce equation," says that groundwater should be extracted in every period until the marginal benefit of extraction is equal to the MC plus a second term $\frac{\dot{p}}{r}$, which is referred to as MUC.[6] The MUC is the loss in PV that would result from an incremental reduction in the resource stock.[7] If equation (1) were not satisfied, there would be an opportunity to increase PV welfare by reallocating pumping over time. For example, if the left-hand side of equation (1) was less than the right-hand side for some period t, welfare could be increased by reallocating excess pumping into the future. That is, the discounted capital gains of leaving some of the groundwater *in situ* would exceed the marginal benefit of pumping that water in period t.

The simple Pearce equation (1) can be extended to the more general renewable case by expanding the MUC term. For a renewable groundwater resource, the "extra benefit of waiting" is determined by how much the net recharge of the aquifer, valued at the net price, changes in response to a marginal increase in the groundwater stock. Now the condition that the marginal benefit of waiting must be equal to the marginal cost of waiting is given by:[8]

$$p_t - c(h_t) = \frac{\dot{p}}{r} + M(h_t) \tag{2}$$

Because both net recharge and unit extraction cost are functions of the head level in this case, they show up in the second component (M) of the MUC expression. There are now two competing effects of extraction: (1) reducing the stock increases future pumping costs, but (2) reducing the stock also reduces SGD and hence increases net recharge. Solving the management problem using the Pearce equation ensures that this tradeoff is accounted for and that the resulting solution maximizes PV. However, as discussed in the following section, using the Pearce equation directly to numerically solve real-world groundwater optimization problems can sometimes be challenging. The remainder of the paper describes a relatively simple alternative approach using Excel's Solver.

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[6] We call this the "Pearce equation" in honor of David Pearce, e.g., Pearce and Markandya (1989). The full Pearce equation is P = c + MUC + MEC, where MEC is the marginal externality cost, for example, the pollution cost of a resource such as coal.

[7] For a demonstration that the lost PV from mining the marginal unit of a nonrenewable resource is $\frac{\dot{p}}{r}$, see Pongkijvorasin and Roumasset (2007).

[8] See the Appendix for a more detailed discussion about the components of $M(h)$.

3.2 Numerical Solution Strategies: Back to the Future, Forward to the Future, and Solver

The problem is now how to operationalize the theory so that students can solve for the optimal extraction path, given aquifer parameters and demand over time. While solution methods for nonrenewable resources and fisheries are well known, for example, the Excel solutions in Conrad (1999), there are no comparable methods for groundwater economics available in any textbook. Even in advanced articles that use the Excel method for groundwater economics, space limitations have precluded a complete exposition suitable for students.

Different algorithms can be devised to solve the groundwater extraction problem described above. In this section, we discuss two options that directly use the necessary conditions for the optimal control approach. The first solution algorithm starts at the future steady state and works back to an ever less distant future. If demand for water is growing over time, we can be sure that a backstop resource, an abundant but costly alternative such as desalination, is eventually implemented in the steady state, and thus the steady state efficiency-price will be $p^* = c_b$, where c_b is the unit cost of desalination. It will then also be possible to solve for the optimal steady state head level h^* by setting $\dot{h}$ and $\dot{p}$ equal to zero in the equation of motion and the Pearce equation (2). We do not, however, know the time T, at which the efficiency price path optimally reaches c_b. Therefore, the first step is to guess T. The Pearce equation with $p_T = p^*$ and $h_T = h^*$ will then tell us the value of p_{T-1} along the candidate optimal path. Once p_{T-1} is determined, h_{T-1} can be generated using the equation of motion. This process can be repeated until $t = 0$ (the initial period), at which point the h_0 generated by the candidate path should be compared to the actual initial head level. If the candidate and actual initial values match, then the solution is optimal. If not, the guess for T must be adjusted, and the process repeated until the initial conditions match.

An alternative approach is to guess the initial efficiency price and iterate forward. With the initial head level h_0 given, a guess for the initial efficiency price p_0 allows us to determine the head level in the next period using the equation of motion, as well as the efficiency price in the next period using the Pearce equation (2). This process can be repeated until the price reaches the optimal steady state price p^*. Recall that we can solve for h^* and p^* by setting $\dot{h}$ and $\dot{p}$ equal to zero in the equation of motion and the Pearce equation. If the head level reaches h^* by the time $p = p^*$, then the initial guess for p_0 was correct, and the candidate paths are optimal. If not, then the guess for the initial price should be adjusted, and the process repeated until the terminal conditions are satisfied.

3.3 An Alternative Approach Using Excel's Solver

For relatively simple groundwater optimization problems, Excel's Solver can be used to find the PV-maximizing paths without using the Pearce equation directly. There are three types of cells required to complete a Solver evaluation: (1) objective cells, (2) variable cells, and (3) constraint cells. In the context of groundwater management, the objective cell contains the PV of water consumption—this is the value to be maximized. The setup should also include two columns of variable cells, one each for the quantity of groundwater pumped and the quantity of desalination, where each row represents a time period, that is, the first row includes control variables for $t = 1$, the second row includes variables for $t = 2$, and so forth. Lastly, a vector of constraint cells should be added that incorporates the equation of motion for the groundwater stock. Note that the initial value for the head level h_0 should be included in the first constraint cell, and an initial guess for candidate solution values should be filled in for the variable cells. When Solver is selected in Excel, a dialog box will pop up asking the user to "set the objective" (objective cell) "by changing variable cells" (variable cells), "subject to the constraints" (constraint cells). Once the appropriate cells are selected, the user should also ensure that "max" rather than "min" is selected and choose one of three solving methods from the pulldown menu. Clicking "Solve" instructs Solver to adjust the values of the control variable cells, subject to the constraint cells, until the PV in the objective cell is deemed a maximum.

Although the numerical solution using Solver is relatively straightforward, there are a few

caveats to keep in mind. First, because the length of the control and constraint vectors are chosen before the optimization takes place, this approach is more suited for finite time horizon problems, that is, problems for which the number of management periods is predetermined. Nevertheless, the infinite time horizon solution can be approximated by rerunning the simulation for longer and longer time horizons until the terminal values are not sensitive to the extension.

3.4 Pearl Harbor Application

Using the PHA on Oahu, Hawaii, as a case study, we provide a step-by-step example of how to apply the Excel Solver method described above. Model assumptions including parameter values are largely drawn from Burnett and Wada (2014). We provide as a supplement to this paper an Excel spreadsheet containing all of the equations and parameters we describe below, along with the maximized solution paths for our two demand cases.

In the case of Pearl Harbor, freshwater can be extracted (q) via pumping wells, but it can also discharge (L) naturally through low permeability caprock (coastal plain deposits) that bounds the freshwater lens along the coast (Figure 1). As the head level (h) declines, the fresh water lens gets thinner and begins to mix with the ocean water below it. At some point, $hmin$, any further extraction would yield water saltier than the environmentally prescribed maximum allowable concentration.[9] Note that since extraction could otherwise continue, thereby increasing net recharge further, net recharge is still a decreasing function of the head level at $hmin$.

We define the period-t benefit (B_t) as the area under the inverse demand curve for water up to the total quantity consumed (Q_t) in period t. The demand for water is modeled as a constant elasticity function: $D(p_t, t) = \alpha e^{gt} p_t^{\eta}$, where g is the growth rate of demand, η is the demand elasticity, and α is a coefficient calculated using actual pumping and price data. The inverse demand curve is then $p_t = ((Q_t/\alpha)e^{-gt})^{(1/\eta)}$. Values for the demand parameters α, η, and g, are assigned to cells B2, B3, and B4, respectively, in the Excel spreadsheet (Figure 2).

The total quantity of water consumed in period t is the sum of the quantity of groundwater extracted (q_t) and the quantity of desalinated water (b_t), that is, $Q_t = q_t + b_t$. When it is optimal to use the backstop resource, total quantity is determined by the backstop price:

$$Q(p_t = p_b) = \alpha e^{gt} p_b^{\eta}, \tag{3}$$

where $p_b = c_b + c_d$. Values for the unit cost of desalination (c_b) and the unit cost of distribution to end users (c_d) are assigned to cells E4 and E5, respectively, in Figure 2. For each period, the total quantity (that would be) demanded at the backstop price is calculated in cells (D9:D108). Following equation (3), in period 1, for example, D9 = \$B\$2 * ((\$E\$4 + \$E\$5) ^ \$B\$3) * (EXP(\$B\$4 * A9)). Note that for the case with zero demand growth, this value remains constant over time. The optimal desalination quantity in each period is determined in cells (E9:E108) according to the following criteria: if $q_t \geq Q(p_b)$, zero desalination is optimal; if $q_t < Q(p_b)$, the optimal quantity to desalinate is $Q(p_b) - q_t$. The formula for period 1, for example, is E9 = IF(B9 > D9, 0, D9 - B9).

Recalling that the total quantity of water demanded is the sum of the quantity of groundwater extracted and the quantity of desalination, we can now go back to the inverse demand curve, which will be used to generate the efficiency price path (C9:C108). As a starting point, the efficiency price in period 1 is C9 = (((B9 + E9) / \$B\$2) * (EXP(-\$B\$4 * A9))) ^ (1 / \$B\$3) in Figure 2. In subsequent periods, the efficiency price changes in accordance with changes in the optimal quantities of extracted groundwater and desalination.

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[9] For the United States, the Environmental Protection Agency sets this limit at 2 percent of ocean salinity.

	A	B	C	D	E	F	G	H	I	J
1	water			cost			head			
2	alpha	107.40		beta	0.00137		R	220		
3	n	-0.25		e	272		h0	17.10		
4	g	0.00		cd	3.39		hmin	15.125		
5	r	0.03		cb	8.46		gamma	78.149		
6	q-cutoff	20								
7										
8	time	q	p	Q (p = pb)	b	C	h	I	hdot	PVNB
9	1	77.2	3.75	57.89	0.00	3.74	17.10	73.40	0.32	5,006
10	2	77.2	3.75	57.89	0.00	3.74	17.42	76.20	0.31	4,858
11	3	77.2	3.75	57.89	0.00	3.74	17.74	78.94	0.30	4,715
…	…	…	…	…	…	…	…	…	…	…
108	100	77.3	3.74	57.89	0.00	3.73	23.84	142.43	0.00	257
109							23.84		TPVNB	160,971

Figure 2: Excel Spreadsheet for the Case of Optimal Groundwater Management with No Demand Growth

The remaining piece to consider in the objective function is the unit cost of groundwater extraction, which is specified as a linear function of lift:

$$c_q(h_t) = \beta(e - h_t) \qquad (4)$$

where e is the ground surface elevation of the well, $e - h_t$ is the the distance groundwater must be lifted to the surface, and β is a coefficient calculated using actual pumping cost and lift data.[10] The extraction cost coefficient and elevation values are specified in cells E2 and E3, respectively, in the Excel sheet (Figure 2). As was the case for desalinated water, we want to include the cost of delivering the pumped water to end users (E4). The unit cost of groundwater, inclusive of distribution, is calculated for each period in (F9:F108). Following equation (4), for period 1, the Excel formula is F9 = $E$4 + ($E$2 * ($E$3 - G9)). Because the extraction cost is a function of the head level, we next have to characterize the change in head level over time.

As previously mentioned, the relatively simple hydrology of the modeled aquifer is represented by an equation of motion that describes how the quantity of the groundwater stock (h) changes over time in response to inflows and outflows of water in every period. Outflow from the aquifer is composed of extraction (what we are trying to solve for) and leakage or SGD. Like unit extraction cost, leakage from the aquifer is a function of the head level in every period, although in this case the relationship is assumed to be quadratic:

$$L(h_t) = 0.24972h_t^2 + 0.022023h_t \qquad (5)$$

Leakage is calculated for every period in (H9:H108). Following equation (5), period 1 leakage, for example, is determined using the formula H9 = (0.24972 * (G9 ^ 2)) + (0.022023 * G9). Inflow is determined by an exogenous rate of recharge (H2). Putting everything together, the change in head level in every period ($\dot{h}$ or "hdot" in Figure 2) depends on the constant rate of recharge, the quantity of groundwater extracted (B9:B108), the amount of leakage (H9:H108), and a head-volume conversion factor (H5):

$$\gamma \cdot \dot{h} = R - L(h_t) - q_t \qquad (6)$$

Following equation (6), the first period $\dot{h}$ value is calculated using the Excel formula I9 = ($H$2 - B9 - H9) * 365 / $H$5. Note that the change is multiplied by 365 to convert annual units to daily values. Finally, the head level in each period $t + 1$ is calculated as the head level in the previous period t plus $\dot{h}$. Thus for $t = 1$, h_2 is calculated using the formula G10 = G9 + I9.

With all of the economic and hydrologic variables and parameters now accounted for, the final step before applying the Solver optimization is to define the objective of the maximization problem. For each period, the present value net benefit (PVNB) is equal to the area under the inverse demand curve (B), net of extraction and desalination costs, discounted at rate r (B5):

$$e^{-rt}\left[B(q_t + b_t) - c_q(h_t)q_t - c_b b_t\right] \qquad (7)$$

Following equation (7), PVNB for period 1, for example, is calculated using the following Excel formula: J9 = ((((((EXP(-$B$4 * A9) / $B$2)) ^ (1 / $B$3)) * (($B$3 / ($B$3 + 1))) * ((B9 + E9) ^ (($B$3 + 1) /

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[10] More generally, the exponent for the lift term need not be equal to one, particularly when considering management of multiple wells simultaneously, because in that case, pumping may be shifting to (more costly) higher elevation wells as the aquifer is drawn down over time. For our purposes, we assume that the need for more wells at higher elevations occurs so rarely that the extraction function is approximately linear.

$B$3) - ($B$6) ^ (($B$3 + 1) / $B$3)) - ((B9 * F9) + (E9 * ($E$4 + $E$5)))) * EXP(-$B$5 * A9))), where the formula for the area under the inverse demand curve has been substituted for B. Note that a minimum quantity or choke price is required to ensure that the area under the demand curve is finite; in this case *q-cutoff* is set at 20 units (B6). The total PVNB or TPVNB over the entire planning horizon is then J109 = SUM(J9:J108).

To solve the groundwater optimization problem, start by opening the "Solver Parameters" dialog box (Figure 3). The TPVNB cell ($J$109) should be set as the objective, and the "Max" option should be selected. Next, Solver requires the user to specify which cells to change to find the solution. For this problem, the objective is to maximize TPVNB by "changing variable cells" $B$9:$B$108 (the groundwater extraction path). For many standard problems, it may be sufficient to check the box "Make Unconstrained Variables Non-negative," but in this particular case, there is a minimum allowable head level (*hmin*), below which pumped groundwater would have unacceptably high salinity levels. Therefore, the following constraint is specified: $G$10:$G$109 > = $H$4. Note that to avoid an undesirable terminal effect where q is very high at $t = 100$ (resulting in $h < hmin$ at $t = 101$), the
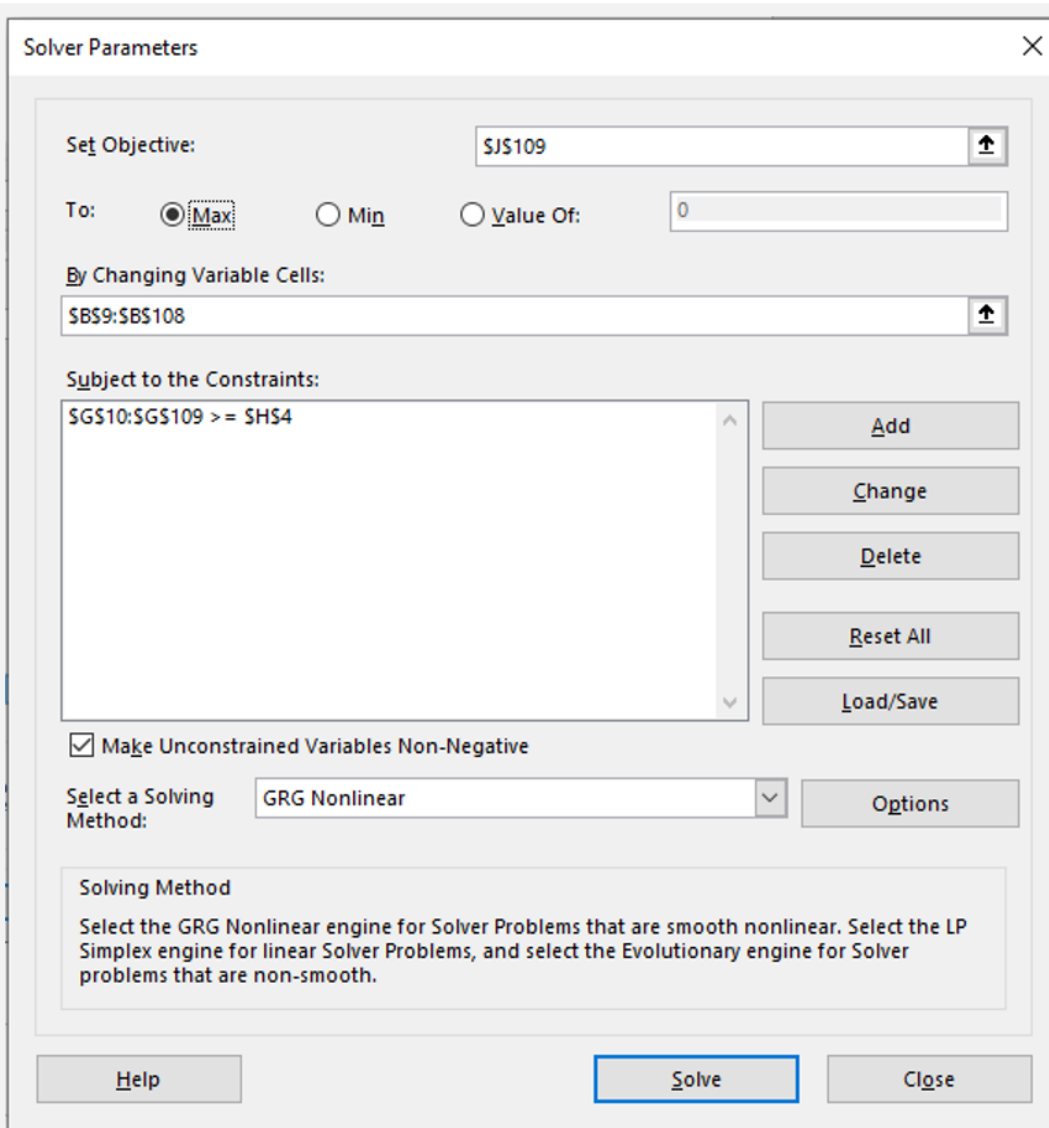

Figure 3: Solver Parameters Dialog Box for the Groundwater Optimization Problem

constraint is maintained from period 2 to 101. As previously discussed, Solver offers a few different solving methods. Here, the GRG Nonlinear method is appropriate. Once all of the parameters are specified, clicking "Solve" generates a solution.[11]

To test the robustness of the simulated results, sensitivity analysis can be performed by adjusting one or more of the parameter values in rows 2–6. The results for the case of 2 percent annual water demand growth are presented in Figure 4. Although the only parameter value adjusted in this case is in cell C4, the entire trajectories of groundwater extraction (B9:B108), efficiency price (C9:C108), desalination (E9:E108), and head level (G9:G108) are different from the no demand growth case in the optimal solution. The optimal paths of these key variables are plotted in Figure 5 for comparison. For the no growth case, the optimal head in the long run is higher than *hmin*, the head level corresponding to MSY, to economize on future extraction costs. Instead of conserving for the purpose of postponing desalination, you are conserving to earn a sustainable dividend from lower extraction costs. Growing demand guarantees that you eventually use desalination, in this case a little after 60 years. The backstop puts a ceiling on the efficiency price. Extraction is constant at MSY in the long run, and demand growth is entirely served by increasing desalination.

In the early (roughly 40) years, optimal extraction is below MSY, allowing the head to increase. Thereafter extraction is above MSY (optimal overdraft) until the head is drawn down to its steady state level (at year 64). From Figure 5, it is clear that the optimal trajectories of *q* and *h* are much different from the cases where extraction is set to maintain the current head level or to the level (*hmin*) that corresponds to MSY.

We can also see from columns C and F in Figure 4 that the efficiency price always exceeds MC along the optimal path, including at the steady state. This means that the common practice of setting *p* equal to extraction cost will induce consumers to waste water (use more than optimal).[12] We encourage teachers and students to get a feel for the optimality of this case as well as the no demand growth solution by trying slight deviations from the optimal exaction path in the Supplementary Excel worksheet (which contains the complete optimal solution for the 100-year time horizon for both cases) to see directly consequences of departing from the optimal path.

4 Extensions to the Basic Coastal Groundwater Management Problem

4.1 Interior Aquifers
The solution for the PHA can be emulated for any coastal aquifer. Since an internal aquifer is a special case of a coastal one, albeit with leakage (SGD) = 0, the Solver approach described can also work for that case. For cases of very low demand growth, however, the number of periods needed for convergence may be impractical. For such cases, the user can redefine the period length, say to 5 years (with the corresponding adjustment of the discount rate).

The spreadsheet-solution method can be also extended to include interlinkages between groundwater and upstream/downstream resources as discussed below.

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[11] For the simple case illustrated below, the solution is not sensitive to the choice of initial values so long as they do not violate a constraint. For example, the method will still work even if initial extraction is set to zero. One strategy is to use the actual extraction rate as the guess for initial extraction.

[12] In the steady state, net recharge is entirely consumed and supplemented with the backstop resource. This means that a royalty is earned on the extracted water, but not from the backstop.

	A	B	C	D	E	F	G	H	I	J
1	water			cost			head			
2	alpha	107.40		beta	0.00137		R	220		
3	n	-0.25		e	272		h0	17.10		
4	g	0.02		cd	3.39		hmin	15.125		
5	r	0.03		cb	8.46		gamma	78.149		
6	q-cutoff	20								
7										
8	time	q	p	Q (p = pb)	b	C	h	L	hdot	PVNB
9	1	78.3	3.83	59.06	0.00	3.74	17.10	73.40	0.32	5,447
10	2	79.9	3.83	60.25	0.00	3.74	17.42	76.15	0.30	5,749
11	3	81.5	3.84	61.47	0.00	3.74	17.72	78.78	0.28	6,067
...	...	...	...	...	...	...	...	...	...	...
108	100	162.5	11.84	427.72	265.18	3.74	15.12	57.46	0.00	822,499
109							15.12		TPVNB	16,727,613

Figure 4: Excel Spreadsheet for the Case of Optimal Groundwater Management with Demand Growing at 2 Percent Annually

Figure 5: Optimal Trajectories for Groundwater Extraction, Desalination, Total Water Consumption, Efficiency Price, and Head Level for the Cases of 0 Percent and 2 Percent Annual Demand Growth

4.2 The Watershed

In the basic groundwater optimization model, we took recharge as exogenously given. More generally, recharge depends on the quality of the upstream watershed. Without maintenance expenditures, the watershed will tend to depreciate, often due to invasive species. Investment in watershed management can maintain a given quality, for example, through fencing and species removal, or improve the watershed condition, say by replanting. This relationship can be simply represented as $R(N)$ where R is the rate of recharge, and N is the quantity of watershed capital. The latter changes over time according to:

$$\dot{N}_t = I_t - \delta N_t \tag{8}$$

where I is the addition to watershed capital via investment and δ is the depreciation rate. The problem is to solve simultaneously for groundwater extraction and investment in watershed capital. The condition for optimal groundwater extraction is prescribed by the same Pearce equation (2) given previously. The condition for optimal watershed investment is again determined by the Nike condition. In this case, "just do it" refers to generating recharge. The

marginal benefit thereof is the "shadow" price of water. Its marginal cost is the cost of increasing N by one unit divided by the marginal recharge from N, that is,

$$\frac{c_I(r + \delta)}{R'(N_t)} = \lambda_t \tag{9}$$

where c_I is the cost of investing in a unit of N, r is the real interest rate, and λ is the shadow price of water given by both its net price and MUC.[13] In this case, extending the basic groundwater optimization problem detailed in the previous section would require values for the recharge function parameters, as well as the depreciation rate and unit cost of investment. Investment in watershed capital for every period would also need to be included with groundwater extraction as control variables for the maximization problem, and the watershed capital itself should be tracked in every period, given that it enters the MUC in the Pearce equation via the recharge function. An example of joint watershed and groundwater optimization can be found in Wada, Pongkijvorasin, and Burnett (2020) for the Kiholo aquifer on Hawaii Island. A similar approach could be taken for the PHA using watershed conservation data from, for example, Bremer et al. (2021).

4.3 Groundwater Dependent Ecosystems

In the case of the isolated aquifer discussed previously, the SGD was important for calculating net recharge but any environmental effects of SGD were not considered. In general, SGD can change the concentration of nutrients in nearshore marine locations (e.g., bays and estuaries), and these effects can be either beneficial or harmful. In particular, SGD tends to lower the salt concentration and can support a valuable ecosystem that thrives in brackish water. On the Big Island of Hawaii, brackish water supports a native seaweed species that is sought after for local dishes (e.g., "poke bowl") and is the foundation for a marine food web that supports invertebrates and other marine life (Duarte et al. 2010). SGD is a decreasing function of groundwater stock or head level. This means that in addition to MUC, groundwater extraction imposes the additional cost of reduced environmental benefits to the nearshore marine ecosystem.

A similar approach can be used to extend the basic groundwater optimization model for Pearl Harbor to include consideration of groundwater dependent ecosystems (GDE). Understanding of two key relationships is required. First, the relationship between SGD (l) and nearshore salinity (s) is given by:

$$s_t = f\big(l(h_t)\big) \tag{10}$$

The second relationship is between nearshore salinity and some biological measure of the valued groundwater dependent ecosystem such as the growth rate of seaweed (x):

$$x_t = w(s_t) \tag{11}$$

If the resource manager's goal is to maintain GDE growth at or above a target level, then a constraint can be added to the standard optimization problem: $x_t \geq \underline{x}$. One can then derive a modified Pearce equation, wherein the MUC term is dependent on how fast x increases with h (the first derivative of equation (11) with respect to h), inasmuch as extraction decisions today affect future head levels, which in turn affect the trajectory of SGD and ultimately the valuable GDE (Wada et al. 2020). The Excel sheet for the standard groundwater optimization problem

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[13] See for example, Roumasset and Wada (2015) for a less heuristic derivation.

(Figure 2) need only be slightly modified. Two columns would need to be added to track salinity (equation 10) and seaweed growth (equation 11) over time, and an additional constraint would need to be specified in the Excel dialog box (Figure 3) to bound seaweed growth from below. In general, if SGD has a beneficial effect on the GDE under consideration, we expect optimal extraction to be somewhat lower, leading to a higher head level in the long run and correspondingly higher SGD.

4.4 Multiple Aquifers

In our base case, we considered a single aquifer with a uniquely corresponding demand for water. In some situations, however, demand may be serviced by multiple aquifers. In that case, the water manager must decide how much to withdraw from each. Drawing from energy economics, we know that optimal extraction from different sources (e.g., oil and coal) is governed by the principle of least-cost-first. Unlike extracting different grades of the same resource, however, full marginal cost (FMC) cannot be ordered by c alone, but by c + MUC.[14] In the case of groundwater, a renewable resource, the MUC for each aquifer is determined by characteristics unique to that aquifer, and the least-cost-first principle requires comparison of the FMCs.

On Oahu, an example is provided by the Honolulu and Pearl Harbor aquifers. Consumption is recorded in the Honolulu and Pearl Harbor "districts" making it tempting to manage the two aquifers independently according to their respective demands. But since the two systems are connected by pipes, this would be a mistake. The PHA has a lower FMC because of its faster leakage. The cost of extracting from the PHA is correspondingly lower because lowering its head level means less SGD flows to the ocean. Thus, the optimal extraction profile is to first extract from PHA until either its MUC rises to equal that of the Honolulu Aquifer (HA) or MSY is reached at the minimum head level. In the application in question, it turns out that the latter comes first. The optimal sequence is then to service the joint demand from only the PHA until head is reduced to *hmin*, where net recharge is also maximized. Thereafter only net recharge is withdrawn from PHA, meaning that the balance of demand is met by the HA, which eventually reaches MSY as well. In subsequent periods, any excess of demand comes from the backstop resource, desalination.

This extension requires separate sets of hydrological parameters characterizing each aquifer. That means two head level trajectories and two extraction paths would have to be tracked over time, where the change in head level (*hdot* in the Excel sheet) for each period is governed by

$$\dot{h}_i = R_i - L_i(h_{it}) - q_{it} \tag{12}$$

for aquifers *i* = 1,2. However, the general solution strategy remains unchanged: vary the control variables (extraction from each aquifer, desalination) subject to the head constraints ($h_{it} \geq hmin_i$ in every period *t* for aquifers *i* = 1,2) to maximize the total PVNB, which in this case includes consumption benefits of water drawn from both aquifers and desalination.[15]

5 Conclusion

We have detailed how renewable resource economics is applied to the problem of welfare maximizing groundwater extraction and shown how to obtain a numerical solution with Excel. The optimal path of groundwater pumping is typically not constant over time in transition to that steady state. In contrast,

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[14] For a single resource, the MUC for different grades is the same. There is only one resource price so $MUC = \dot{p}/r$ is the same for different grades. Therefore, the principle of least FMC degenerates to the Herfindahl condition of least-extraction-cost-first. For different resources, which have different resource prices, that does not follow, and we need to use FMC instead (Chakravorty and Krulce 1994). In the case of water, there is only one water price, but the MUCs of different aquifers are typically different.

[15] We recommend using either using actual values for initial guesses or obtaining independent solutions for the resources and using those to set the initial guesses for the combined resource system.

management strategies that specify a target pumping constraint (safe yield, sustainable yield, etc.) are incomplete, in the sense that they usually do not describe the approach path, which may be fast, slow, or even nonmonotonic. The method also provides the corresponding paths of efficiency prices. By equating marginal prices to efficiency prices, a water authority can implement the efficient extraction program.[16]

In the illustration for PHA, optimal extraction, for the base case of 2 percent demand growth, is well below MSY for most of the first (roughly) 40 years, more than MSY for the next (about) 24 years, and then equal to MSY beyond that. Not only does MSY deplete the aquifer too fast, but it squanders the potential gains from early conservation. The results are even stronger for the case of zero demand growth. Optimal extraction is constant over time and well below MSY, implying large welfare losses from extracting at the MSY level.

Through the basic groundwater management example presented here, along with the possible extensions such as watershed conservation, protection of groundwater dependent ecosystems, and the management of multiple aquifers, it should be clear that while optimal water management is sustainable, management strategies that primarily aim to sustain the resource at a desired level are not likely to be optimal. It should be noted that while the examples presented in this paper focus on characteristics of the groundwater resource and direct water consumption, incorporating additional environmental and socioeconomic considerations does not change the conclusion that optimal water management is sustainable but not the other way around.

The solution method detailed and illustrated here can also be adapted to inland aquifers. The archetypical inland aquifer (e.g., Burt 1966) is a special case of a coastal aquifer, wherein discharge is zero. Moreover, recharge is constant except when the aquifer is so near its maximum capacity that it cannot absorb the full amount of recharge. For this case, both discharge and recharge growth can be suppressed so long as the absorptive capacity is not a limiting condition in the optimal solution. While this sounds very much like a nonrenewable resource, some components of the M term in equation (2) do not approach zero. This means that the efficiency price remains above the extraction cost even in the long run. While students should be able to navigate the Excel spreadsheet provided in the supplementary materials, we recommend the instructor demonstrate the example provided in the chapter with the selected parameters as an in-class exercise as a starting point. Students can then try alternative formulations developed by the instructor as take-home exercises, including the extensions described in Section 4. While the tool is designed to allow for such extensions, adding multiple state or control variables to the application may require additional Excel add-ins not required to run the basic groundwater model.[17] We recommend that instructors test extensions such as linked watersheds, aquifers, or marine habitat impacts with the standard version of Excel to investigate which add-ins may be needed before assigning these more complex models to students. Even without students engaging in these more complex exercises, it will be useful for them to see that solving for optimal extraction from an isolated aquifer is at best a first approximation of the solution when groundwater is part of an ecological system.

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[16] Since efficiency prices are above extraction costs, charging the efficiency price for all consumption units will generate a revenue surplus. One way to return the surplus to consumers is by charging inframarginal block prices below the FMC.

[17] These Excel add-ins provide optional commands and features for Excel. By default, all add-ins are not immediately available in Excel but typically many are free, especially with the standard university licenses. Add-ins require installation and in some cases activation, typically from the File/Options menu depending on the version.

Appendix: Maximization Problem and Optimality Conditions

Mathematically, the dynamic optimization problem is as follows:

$$max_{q_t,b_t} \int_0^\infty e^{-rt}\big[B(q_t + b_t) - c_q(h_t)q_t - c_b b_t\big]\, dt \qquad (A1)$$

where B denotes the benefits of water consumption, and c_q and c_b are the unit costs of groundwater pumping (q) and a backstop resource (b) such as desalination, respectively. For those without a calculus background, this may look scary, but it is just a way of adding up the discounted benefits and costs from now on. The water manager may choose to supplement groundwater pumping with desalination, an abundant but costly alternative called a *backstop* resource. Note the subscript t for the variables q and b, which indicates that the solution requires selecting values for the choice variables (also known as "control variables") in every period until the end of the planning horizon. That is, the planner must choose the complete approach paths (i.e., not just values for the current period) for q and b that maximize PV. The discount factor (e^{-rt}) converts the net benefits accrued for each period t into PV terms.

While there are an infinite number of candidate sequences that can potentially solve the maximization problem (A1), we can narrow down the possibilities by imposing an *equation of motion* for the aquifer stock. For a single-cell coastal aquifer, the change in groundwater stock over time is described by the following equation:

$$\gamma \cdot \dot{h} = R - L(h_t) - q_t \qquad (A2)$$

The head level (h) or distance from mean sea level to the top of freshwater lens is an index for groundwater volume. The change in head level over time, denoted as $\dot{h}$, is converted to change in volume by a constant conversion factor γ. Recharge (R) is assumed exogenous, and leakage (L) out of the aquifer via SGD is a function of the head level. In the discussion that follows, we suppress γ for simplicity and interpret $c_q'(h_t)$ as the change in unit cost per volumetric unit of groundwater stock.

We start with the simplification wherein SGD is zero (e.g., an inland aquifer), and the recharge rate is effectively zero over the management horizon of interest. In this case, groundwater can be treated as a nonrenewable resource. The marginal benefit of waiting is $\dot{p}$, that is, the capital gain from selling at next period's price. The marginal cost of waiting is the foregone interest from harvesting today instead of one period later. Extraction is optimal when the marginal benefit is equal to the marginal cost of waiting:

$$p_t = c(h_t) + \frac{\dot{p}}{r} \qquad (A3)$$

Equation (A3), called the "Pearce equation," says that groundwater should be extracted in every period until the marginal benefit of extraction, $p_t \equiv B'(q_t + b_t)$, is equal to the MC plus a second term $\frac{\dot{p}}{r}$, which is referred to as MUC. The MUC is the loss in PV that would result from an incremental reduction in the resource stock. If equation (A3) were not satisfied, there would be an opportunity to increase PV welfare by reallocating pumping over time.

The simple Pearce equation (A3) can be extended to the more general renewable case by expanding the MUC term. For a renewable groundwater resource, the "extra benefit of waiting" (postponing the marginal unit extracted until next period) is: $\frac{d}{dh}[(p_t - c_q(h_t))F(h_t)]$, where $F(h_t) \equiv R - L(h_t)$ is the net recharge function (more generally the net growth function of the resource). Now

the condition that the marginal benefit of waiting must be equal to the marginal cost of waiting is given by:[18]

$$p_t - c_q(h_t) = \frac{\dot{p}}{r} + \frac{[p_t - c_q(h_t)]F'(h_t)}{r} - \frac{c_q'(h_t)F(h_t)}{r} \qquad (A4)$$

Because both leakage and unit extraction cost are functions of the head level in this case, they show up in the MUC term. The second term on the right-hand side of (A4) is negative because $F'(h_t) < 0$. That is, there is actually a benefit of extraction because of the increased net recharge. The third term is positive because the higher the water table, the lower the unit extraction cost, that is, $c_q'(h_t) < 0$. Note that the first term in the MUC goes to zero ($\dot{p} = 0$) as we approach the steady state solution in the long run, but that the entire MUC can remain positive.

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[18] This follows Pearce and Turner's (1990, p. 255) derivation for a generic renewable resource. Equation A4 can also be rewritten as: $p_t = c_q(h_t) + \frac{\dot{p} - [R - L(h_t)]c_q'(h_t)}{r + L'(h_t)}$. See Burnett and Wada (2014) for a detailed derivation.

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